



Paper Type: Original Article

The Golden Ticket Searching the Impossible Fractal Geometrical Parallels to Solve the Millennium, P vs. NP Open Problem

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Citation:

Received: 07 March 2024

Revised: 24 May 2024

Accepted: 01 October 2024

Mageed, I., & Li, H. (2025). The golden ticket searching the impossible fractal geometrical parallels to solve the millennium, P vs. Np open problem. *Computational engineering and technology innovations*, 2(2), 81-87.

Abstract

The two major unresolved issues in current mathematics and computer science addressed in this paper are fractal geometry and the P vs. NP problem. Benoît Mandelbrot's pioneering work in fractal geometry provides a mathematical basis for comprehending the complex, self-similar forms common in nature defined by non-integer dimensions. Its uses range from computer graphics to erratic systems across several spheres. On the other hand, the P versus A Millennium Prize Problem, NP problem answers the basic question of whether issues with quickly verifiable answers may also be effectively resolved. Emphasizing their status as important frontiers of knowledge, this article explores the basic ideas of both fields, their current knowledge, and the severe consequences their eventual resolution would have on scientific investigation and technology development.

Keywords: Fractal geometry, The P vs. NP problem.

1 | Introduction

Scientific enterprise is defined by the quest of basic truths and the solving of difficult problems. In mathematics and computer science, some open problems are major obstacles since their resolutions might lead to new paradigms of knowledge and competence. This study makes two such issues—the continuous investigation within fractal geometry and the deep P vs. NP problem [1], [2]. Seemingly unrelated, both fields investigate the essence of complexity: one depicting the intricate complexity of natural forms, the other grappling with the intrinsic limits of computational complexity. The basis of these two important fields of study will be discussed in this paper together with their ongoing relevance and present state of knowledge.

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 <https://doi.org/10.48314/ceti.vi.51>

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2| Fractal Geometry: Discovering the Complexity of Nature

largely ascribed to Benoît Mandelbrot [3], [4], fractal geometry is a ground-breaking idea that offers a strong mathematical language for characterizing the irregular, broken, and often infinitely detailed forms seen throughout the natural world, as depicted in *Fig. 1*. [5]. Unlike conventional Euclidean geometry, which applies only to integer-dimensional entities (Such as lines, planes, cubes), fractal geometry [6] welcomes items defined by self-similarity and often has a fraction or fractal dimension.

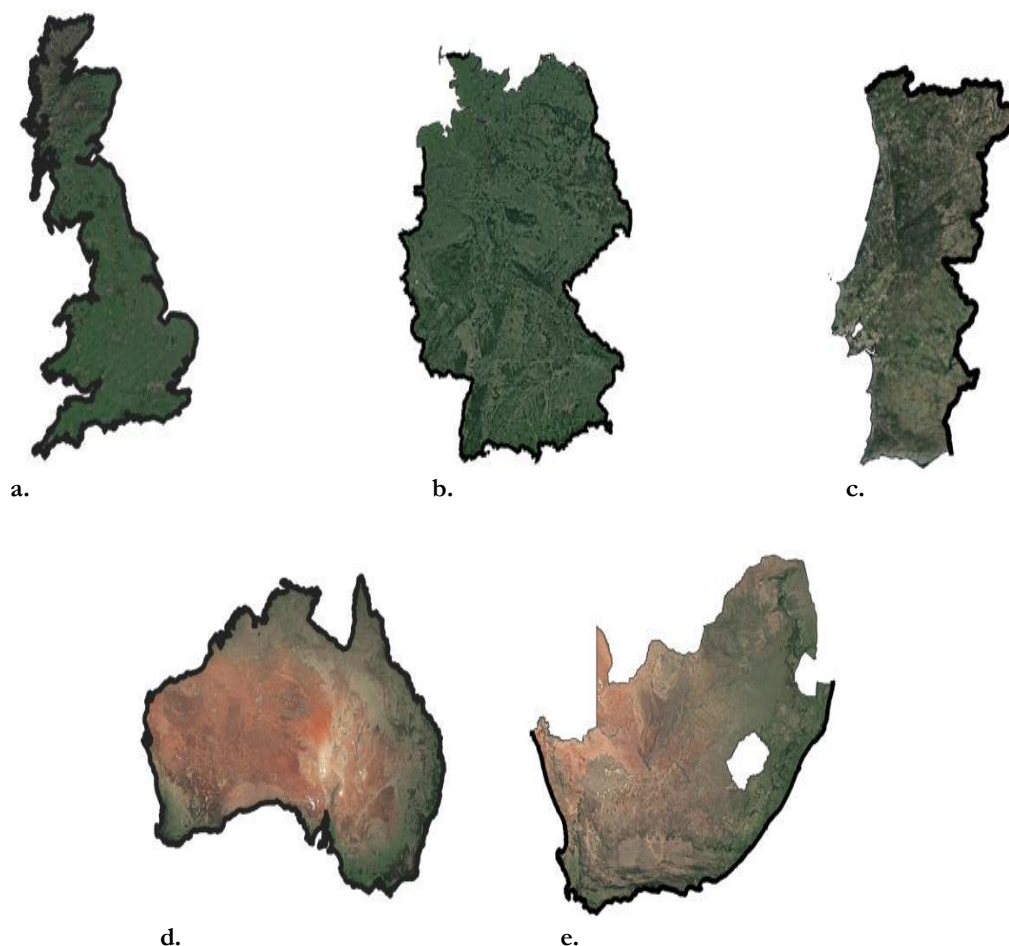


Fig.1. The great britain coastline, the german land border, the portuguese land border, the australian coastline, and the south african coastline are the locations mandelbrot cited; a. coastline of great britain, b. land frontier of germany, c. land frontier of portugal, d.coastline/border of australia, e. coastline of South africa.

2.1| Fundamental Ideas

Self-similarity is the distinguishing feature of a fractal; hence, when enlarged, a portion of the object [7] resembles the entirety. As seen in mathematical constructions like the Mandelbrot set or the Koch snowflake [7], this property can be precise; as seen in natural events, it can be statistically. Crucially, the idea of fractal dimension defines how entirely a fractal occupies space as its scale varies. Unlike topological dimension—which is always an integer—the fractal dimension can be a non-integer, reflecting the object's complex detail and roughness across all scales [8]. Common approaches for estimating fractal dimension are the box-counting dimension and Hausdorff dimension [7].

2.2| Natural Occurrences and Uses

Nature makes extensive use of fractals [9]. Among the examples are the branching patterns of trees, river networks, and blood vessels; the complex structure of coastlines and mountain ranges; the formation of

snowflakes; even the distribution of galaxies [9], [10]. Apart from defining natural shapes, fractal geometry has seen great use in many scientific and technical fields. Fractals are employed in computer graphics to produce realistic landscapes, textures, and special effects [10]. Particularly in chaos theory, where sensitive dependence on initial conditions causes unpredictable, yet often fractally structured, results [11], they are also employed in signal processing, data compression (e.g., fractal image compression), and the analysis of complex systems. From neuron branching to protein folding [12] the study of fractals has also helped to clarify biological systems.

2.3 | Unresolved Issues in Fractal Geometry

Though there have been notable developments, some unresolved issues remain in fractal geometry [13]. These comprise a more profound conceptual grasp of the dynamics of complicated fractal sets [11], especially those generated from iterative functions in higher dimensionalities. Research on the creation of more robust and computationally efficient approaches for reliably calculating fractal dimensions from noisy, real-world data is currently underway [5]. Moreover, the full scope of fractal uses in cutting-edge sectors including customized medicine, material science, and network analysis still being investigated [13]. Additionally offering opportunities for future research is the interaction between information theory and fractal geometry [15].

3 | The P Versus NP Problem: The Computation Frontier

Designated by the Clay Mathematics Institute as a basic challenge in mathematics, one of the seven Millennium Prize Problems is the NP problem with a one million award for its resolution [15]. It addresses the efficiency limits of algorithms and is central to computational complexity theory [15], as in *Fig.2* [17].

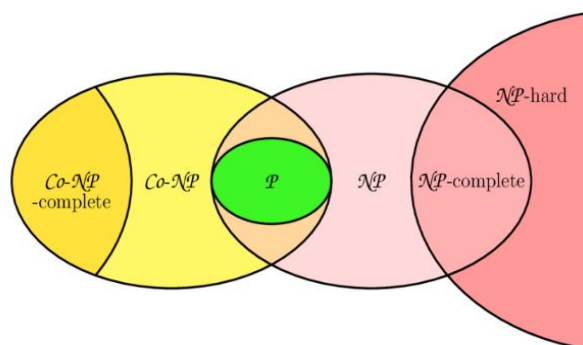


Fig. 2. A few classes of complexity ($P = NP$ is assumed).

4 | Fractal Geometry: Discovering the Complexity of Nature

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5 | The P versus NP Problem: The Computation Frontier

P vs. NP Designated by the Clay Mathematics Institute as a basic challenge in mathematics, one of the seven Millennium Prize Problems is the NP problem with a one million award for its resolution [15]. It addresses the efficiency limits of algorithms and is central to computational complexity theory [15]. Though different in their immediate aim, these two domains basically highlight the extent of uncharted intellectual landscape [5]. They capture the spirit of mathematical and scientific investigation [8], in which basic problems remain unsolved, motivating many years of scientists to expand the frontiers of knowledge and search for the elusive answers that promises to transform our perspective of complexity, nature, and computation.

Often referred as one of the Millennium Prize Problems [15], the P versus NP problem is among the most important unresolved issues in theoretical computer science and mathematics. At its heart, the question asked is if every issue whose answer can be easily checked can also be easily discovered by a computer [16]. This basic question probes the very boundaries of efficient computation.

Decision problems within the class P that a deterministic Turing machine can solve in polynomial time are, these are thought of as tractable or efficiently solvable issues [16]. Sorting a list of numbers or doing simple arithmetic operations, for instance, belong under this category [15]. Problems in P are feasible for larger inputs as their time complexity grows according to a polynomial function of the input size [18].

Conversely, the class NP encompasses decision issues for which a deterministic Turing 1 machine, [15] can confirm a suggested answer in polynomial time. The N stands for nondeterministic, which is the hypothetical capacity of a nondeterministic Turing machine to guess the correct answer and then verify it quickly [13]. A traditional example is the Boolean Satisfiability Problem (BSP), where, given a logical formula, one can easily check whether a particular assignment of truth values satisfies it; nevertheless, discovering such an assignment may be really challenging [15] Other well-known NP challenges include the Traveling Salesperson Problem (TSP) and the Sudoku puzzle, in which finding a solution is simple relative to [19].

Since every problem solvable in polynomial time can surely be verified in polynomial time, $P \subseteq NP$ is broadly agreed upon [16]. Whether $P=NP$ or $P \neq NP$ is the decisive issue. Most mathematicians and computer scientists vehemently think $P \neq NP$ [16]. According to this generally accepted theory [18], there are issues for which proof is naturally simpler than discovery, therefore suggesting basic computational restrictions. Should it turn out to be true, it would show that some issues are inherently difficult independent of technical progress.

$P=NP$ would have massive consequences. Many presently intractable issues—like breaking contemporary cryptographic systems like RSA—would be implied solved efficiently [22], [20]. From artificial intelligence, maybe permitting automated theorem proving and creative breakthroughs, to optimization issues in logistics and medicine design, this would revolutionize disciplines enabling effective answers to difficult real-world problems [19]. On the other hand, it might also undermine several existing security systems.

On the other hand, proving $P \neq NP$ would confirm the bases of current cryptography and our knowledge of computational complexity [20]. It would validate that some computational problems are difficult, hence requiring heuristic solutions for numerous optimization problems [21]. The P versus NP problem, driven by major study on complexity theory and the nature of computation itself, NP problem remains one of the most difficult problems in computer science.

6 | Conclusion

Fundamentally, this paper provides a novel fractal exploratory to establish the solution of one of the most complicated Millenium open problems, ever existed. By default, this would be a phenomenal, informed research-based line of inquiry to unlock the P vs NP mystery. Future research includes exploring fractal links with other Millenniums, in pursuit to start the journey of solving the Millenniums one by one.

Acknowledgments

The authors would like to express their sincere appreciation to colleagues and members of the scientific community whose discussions and constructive feedback contributed to the development of the ideas presented in this study. The authors also acknowledge the pioneering contributions of researchers in fractal geometry and computational complexity theory, whose foundational work inspired this research.

Conflicts of Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data Availability

No new datasets were generated or analyzed during this study. All information used in this research is derived from published literature and publicly available scientific sources. Additional materials supporting the findings are available from the corresponding author upon reasonable request.

Authors' Contributions

I. A. M.: Conceptualization, Methodology, Formal Analysis, Investigation, Writing—Original Draft, Visualization.

H. L.: Validation, Supervision, Critical Review, Writing—Review & Editing.

Both authors have read and approved the final version of the manuscript.

Funding

This research received no external funding.

Consent for Publication

All authors have reviewed and approved the final manuscript and consent to its publication.

Ethics Approval and Consent to Participate

This study did not involve human participants, animals, or identifiable personal data. Therefore, ethical approval and informed consent to participate were not required in accordance with institutional and international research ethics guidelines.

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